NWSC/CR/RDTR-10

ASSURANCE LIMITS FOR VULNERABLE AREA

DATA ANALYSIS TECHNIQUES OF VULNERABILITY ASSESSMENTS

PARTIAL FULFILLMENT OF FY 75
ASSESSMENT QUANTIFICATION PANEL
STATEMENT OF WORK

JERRY KEMP
NORMAN PAPKE
DAVID MONTGOMERY

PREPARED BY

APPLIED SCIENCES DEPARTMENT

NAVAL WEAPONS SUPPORT CENTER, CRANE, INDIANA

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Most vulnerable area (A_v) analyses consider component probability of kill given a hit, P(K/H), to be a point estimate with no variance. This report presents statistical methods for calculating assurance limits for A_v assuming P(K/H)'s are not known with certainty.

(Continued on reverse)

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Normal approximation and Monte Carlo techniques are presented. The accuracy of these techniques is determined analytically for a target with a small number of critical components and extended to a larger number of components by heuristic arguments.

The central limit theorem indicates that A_V will be approximately normally distributed.

Formulas for necessary calculations are shown.

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ABSTRACT

Most vulnerable area (A_v) analyses consider component probability of kill given a hit, P(K/H), to be a point estimate with no variance. This report presents statistical methods for calculating assurance limits for A_v assuming P(K/H)'s are not known with certainty.

Normal approximation and Monte Carlo techniques are presented. The accuracy of these techniques is determined analytically for a target with a small number of critical components and extended to a larger number of components by heuristic arguments.

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Formulas for necessary calculations are shown.

TABLE OF CONTENTS

Page

Abstract i

I. Introduction 1

II. Statistical Analysis and Discussion 2

A. Likely P(K/H) Distributions 3

B. Relevance of Cumulative Distributions . . . 4

C. Argument for Normal Approximation 6

1. Central Limit Theorem 6

2. Coding of the Random Variable, A_i $P(K/H)_i$ 7

3. Mathematically Derived Cumulative for Non-Normal 9

4. Comparison of Non-Normal Mathematically Derived Cumulatives and Their Normal Approximations 11

5. Extensions of Normal Approximations to Other Non-Normal Distributions . . . 23

6. Departure of A_v from Assumptions of the Central Limit Theorem 25

D. Methodology (A_v Approximated by Normal) . . 31

III. Monte Carlo Simulation Description 34

IV. Monte Carlo Simulation Results 37

V. Conclusions 39

References 43

ASSURANCE LIMITS
FOR VULNERABLE AREA

I. INTRODUCTION

In the draft AFFDL Technical Report, "An Objective Confidence Level Methodology for Probability Estimate Values in Survability/ Vulnerability Assessments" (reference (1)), Captain Eugene Steadman cites the need for establishing the cumulative distribution function for vulnerable area (A_v). The standard definition of A_v is

$$A_v = \sum_{i=1}^{n} A_i P(K/H)_i$$

where n is the number of components, A_i is the presented area of the i^{th} component, and $P(K/H)_i$ is the probability of kill given a hit of the i^{th} component by the particular threat, subject to the shotline survivor rule when applicable. In this study, $P(K/H)_i$ is a random variable which has some probability density function.

Statistical methods are explored which could be used to calculate assurance limits for vulnerable area, assuming P(K/H) is not known with certainty. P(K/H) values currently used in vulnerable area analyses are point estimates assumed to have no variance. Through the use of Monte Carlo simulations and statistical analysis techniques, suitable methods for using distributed P(K/H) values in vulnerability analyses are investigated considering feasibility, accuracy and cost effectiveness. The effects that different types of P(K/H) distributions have on the resulting vulnerable area distribution are examined.

A Monte Carlo technique was developed to simulate the cumulative distribution function of A_v, as suggested in reference (1). This simulation is based on the assumption that all $P(K/H)_i$'s are independent;

i.e., all covariances equal zero. This assumption is made not only in the Monte Carlo simulation but also in the analytic techniques. If the covariances are <u>not</u> zero, the probability density function $P(K/H)_2$ must be conditioned (changed) for each selected value of $P(K/H)_1$, the probability density function of $P(K/H)_3$ must then be conditioned for each $P(K/H)_1$ and $P(K/H)_2$, and so forth until $P(K/H)_n$ is a function of the selected values of $P(K/H)_1$, $P(K/H)_2$, ... $P(K/H)_{n-1}$. Methodology for dependent P(K/H)'s exists and is workable for both the normal approximation and Monte Carlo techniques.

The possibility that A_v is distributed normally is explored, since the cumulative normal is tabled to various degrees of accuracy in many reference books. If A_v is normally distributed (or approximately so), these results could be used for analyses in lieu of conducting a Monte Carlo simulation for each individual analysis performed.

The central limit theorem gives an indication that it is very likely that A_v will be approximately normally distributed.

The following sections of this report will deal with a statistical analysis and discussion which examines various possible P(K/H) distributions, a discussion of the Monte Carlo simulation used to test these distributions, and the recommendations and conclusions arrived at as a result of this study.

II. STATISTICAL ANALYSIS AND DISCUSSION

In this section the appropriateness of using a normal approximation for the distribution of A_v will be pursued. The distribution of A_v is formed from the sum of random variables, each associated with its own distribution. Determination of the error made by approximating the distribution of the sum of n random variables by a normal distribution requires determining the actual distribution of the sum. For small n, this can be done analytically; for large n, one must resort to one of several approximate techniques, such as numerical evaluation of convolution integrals or Monte Carlo simulations. In this study distributions were combined analytically for small n and extended to large n by heuristic arguments. Monte Carlo simulations were run for small n to test the model and to measure the accuracy of the simulation for cases where it was possible to analytically calculate the distribution of the sum. The case for large n will be investigated using actual aircraft data and be presented in a later report.

If A_v is normally distributed, the cumulative normal that is tabled to various degrees of accuracy in many reference books could be used for all analyses in lieu of conducting a Monte Carlo simulation for each analysis performed.

A. Likely P(K/H) Distributions

Some P(K/H) values used in vulnerability analysis are arrived at by a rather complicated, though not codified, series of steps involving engineering experience and judgment. In these cases, it is not obvious exactly what distribution would be associated with the P(K/H) value; however, a reasonable approach to this situation would be the assignment of upper and lower bounds (with probability zero) and a most likely (modal) value. This forms a triangular distribution. Other distributions considered are uniform and normal.

Use of a uniform distribution is present in the computation of the probability of rendering a rod non-functional given that the rod is hit (references 4 and 5). An implicit assumption in these references is that all shotlines that hit the rod are equally likely.

B. Relevance of Cumulative Distributions

The long range objective of this effort is to provide a means of determining the most narrow limits (error limits) around the calculated A_v that will provide 95 percent assurance that the actual A_v is within these limits. Translating this objective into statistical terms implies calculating A_v many times (for the same target) considering the distributions associated with the P(K/H) values. (NOTE: This repetitive calculation will not be required in practice; it is merely an artifice used in explaining the following work.) These vulnerable area estimates are then ranked and the limits, those values between which 95 percent of the estimates appear, can be found, and thus the error limits determined. If the A_v estimates are plotted, they form some probability density function for A_v, illustrated in Figure 1.

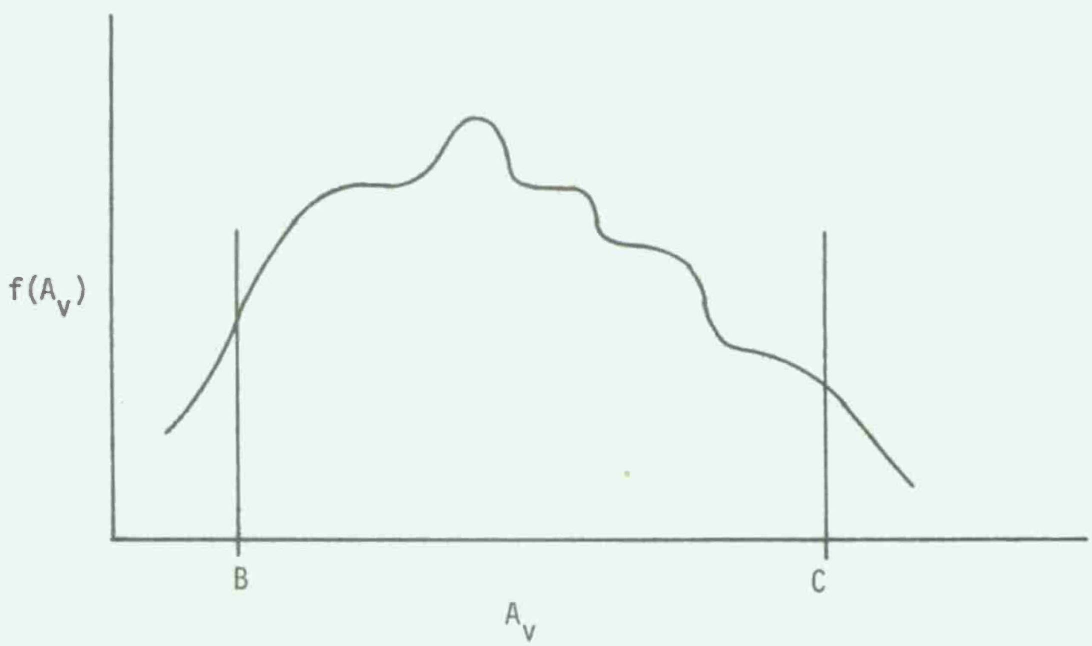


FIGURE 1

The area under the entire probability density function curve is one. That is,

$$\int f(A_v)dA_v = 1$$

The area under this curve between two limits, such as B and C, represent the fraction of the A_v estimates between the limits. That is,

$$\begin{matrix}\text{Fraction of} \\ A_v \text{ estimates} \\ \text{between B and C}\end{matrix} = \int_B^C f(A_v)d(A_v)$$

Since this is of major interest, the analysis will be performed using the cumulative distribution functions, $F(A_v)$.

$$F(A_v) = \int_0^{A_v} f(A_v')d(A_v'),$$

where A_v' is a dummy variable used for integration purposes. Thus the fraction of A_v estimates less than B is F(B) and

$$F(B) = \int_0^B f(A_v)d(A_v)$$

Similarly, the fraction of estimates below C is F(C) and

$$F(C) = \int_0^C f(A_v)d(A_v)$$

Therefore, the fraction of A_v estimates between B and C is F(C) - F(B). This is illustrated in Figure 2.

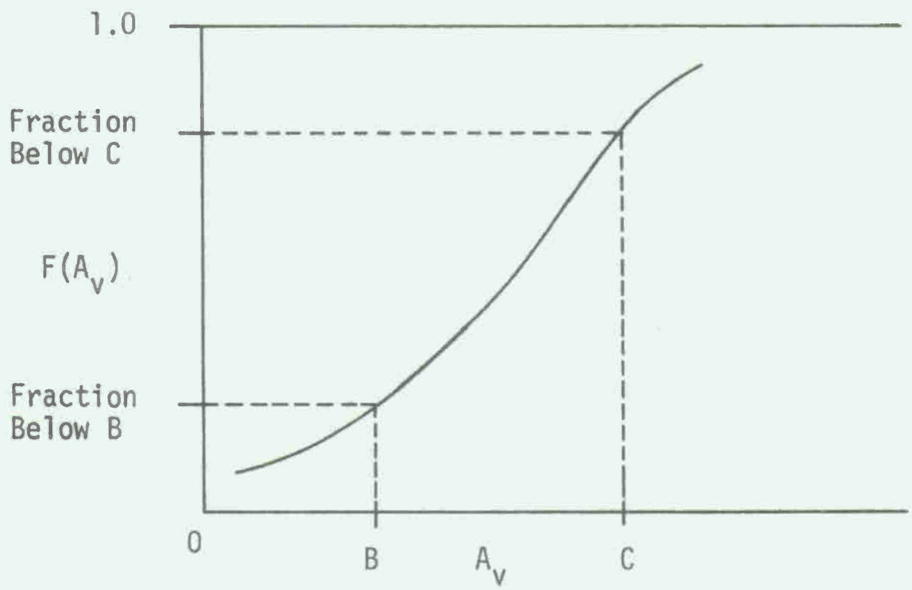


FIGURE 2

C. Argument for Normal Approximation

1. Central Limit Theorem

The Central Limit Theorem (reference 2), loosely stated, says that the distribution of a random variable formed as the sum of n identically distributed random variables approaches a normal distribution. The importance of the Central Limit Theorem to the vulnerable area problem is clearer when the definition of vulnerable area is repeated.

$$A_v = \sum_{i=1}^{n} A_i \, P(K/H)_i$$

$$= A_1P(K/H)_1 + A_2P(K/H)_2 + \ldots + A_nP(K/H)_n$$

The $A_iP(K/H)_i$'s are the random variables considered in the Central Limit Theorem, each variable having some associated distribution; although in most analyses, the distribution has been neglected, and, in fact, may not be known.

The accuracy of the Central Limit Theorem will be investigated in the following paragraphs by considering the departure from normality of distributions formed using sums of random variables with identical symmetrical distributions. Although these initial comparisons are not realistic for an aircraft vulnerability analysis, they serve as the starting point for the investigation. Extensions of these concepts to examples pertinent to vulnerability analysis are discussed in later paragraphs.

2. Coding of the Random Variable, $A_i P(K/H)_i$

For ease of calculation, certain transformations will be performed. The distribution to be considered for comparison of analytically derived results with those obtained using the Central Limit Theorem is shown below in Figure 3. The choice was dictated by convenience in performing the analytic calculations as well as the fact that this is a logical approximation to use when only very limited data is available.

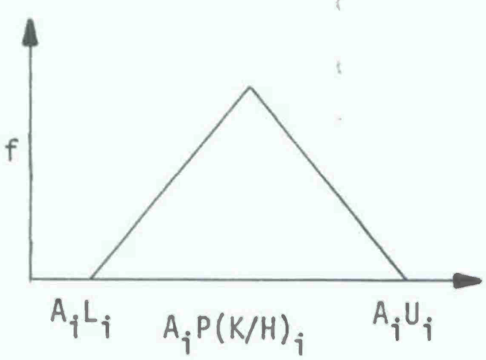


FIGURE 3

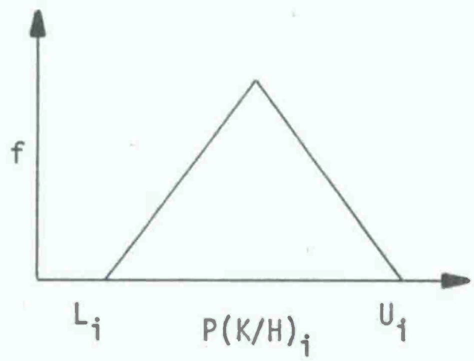


FIGURE 4

In Figure 3, A_i is a constant and $P(K/H)_i$ is a random variable with a lower limit of L_i and an upper limit of U_i.

Dividing by the constant A_i yields the distribution shown in Figure 4.

Further coding to the random variable $P'(K/H)_i$ with values over the the interval 0 to 1 is shown by the following formula and Figure 5.

$$P'(K/H)_i = \frac{P(K/H)_i - L_i}{(U_i - L_i)}$$

$$\text{or } P(K/H)_i = (U_i - L_i)\, P'(K/H)_i + L_i \qquad (1)$$

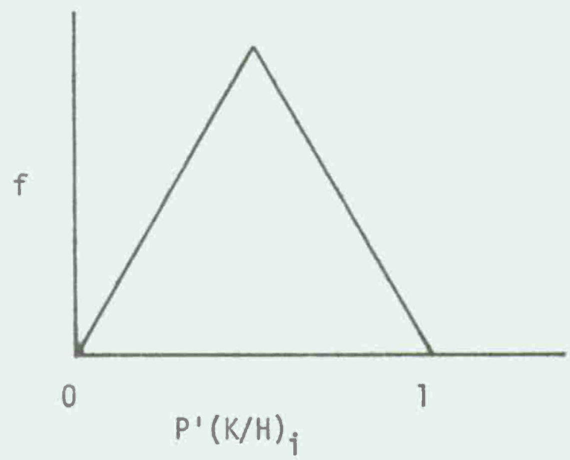


FIGURE 5

Applying Equation (1) to the definition of vulnerable area, A_v can be written as a function of the random variable $P'(K/H)_i$ and associated constants, A_i and $(U_i - L_i)$.

$$A_v = A_1\, [(U_1 - L_1)\, P'(K/H)_1 + L_1] + A_2[(U_2 - L_2)\, P'(K/H)_2 + L_2] + \ldots$$
$$\ldots + A_n[(U_n - L_n)\, P'(K/H)_n + L_n] \qquad (2)$$

Tentatively assume all $A_i(U_i - L_i)$'s are equal. This assumption will be re-examined later. Now A_v becomes

$$A_v = A_1(U_1 - L_1) \sum_{i=1}^{n} P'(K/H)_i + \sum_{i=1}^{n} A_i L_i \qquad (3)$$

Then, set $C_1 = A_1(U_1 - L_1) = A_2(U_2 - L_2) = \ldots = A_n(U_n - L_n)$ and $C_2 = \Sigma A_i L_i$ in Equation (3). It follows that

$$A_v = C_1 \Sigma P'(K/H)_i + C_2 \qquad (4)$$

Using Equation (4), it can be shown statistically that $\Sigma P'(K/H)_i$ and A_v will have the same distribution shape (form). Multiplication by a constant changes the spread of values and addition of a constant changes the location of the values but neither changes the shape nor the form of the distribution.

3. Mathematically Derived Cumulative for Non-Normal

The exact analytic cumulative distribution functions for three non-normal variables are presented in this section. In the next section, an evaluation of using the normal to approximate these exact analytic functions will be made. The three non-normals selected are: (1) a one-variable, symmetric triangular distribution; (2) the sum of two variables, each from identical symmetric triangular distributions; and (3) the sum of twelve variables, each from identical uniform distributions. The variable S (for sum) and the variable A_v (Vulnerable Area) are identical.

The analyses for all three non-normal distributions will be performed using the coded variables, $P'(K/H)_i$, which are distributed over the interval 0 to 1.

a. One Variable - $S_{1,\Delta}$

This random variable ($S_{1,\Delta}$) represents the vulnerable area for the trivial case of an aircraft with only one critical component.

$S_{1,\Delta}$ is a single sample from a symmetric triangular distribution and is defined over the interval (0 to 1). Coding to realistic limits involves the area of the component, and optimistic and pessimistic limits for P(K/H) which were discussed earlier.

The probability density function, $f(A_v)$, for $S_{1,\Delta}$ is shown in Figure 6 below.

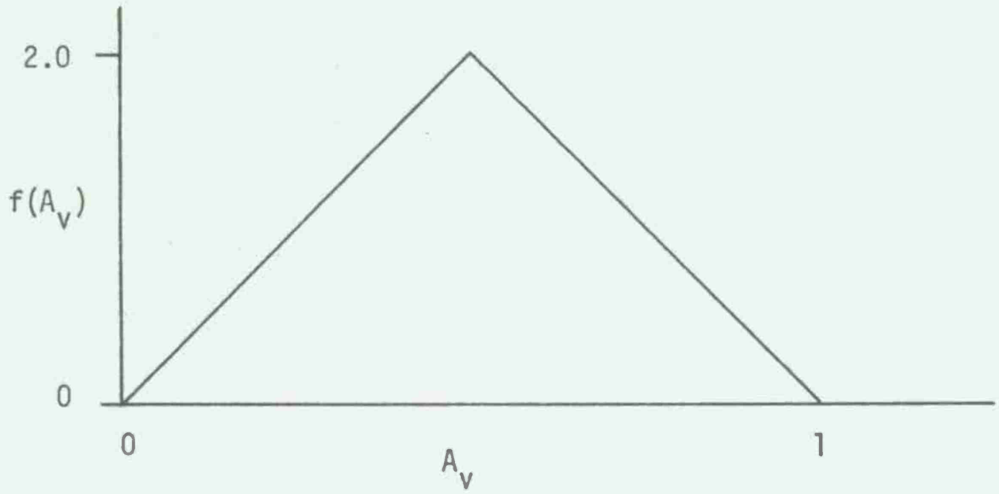


FIGURE 6

The cumulative distribution [area under $f(A_v)$] is defined in the following manner:

$$F_{1,\Delta}(A_v) = \begin{cases} 2(A_v)^2 & , 0 \leq A_v \leq 0.5 \\ -2(A_v)^2 + 4(A_v) - 1 & , 0.5 \leq A_v \leq 1.0 \end{cases} \quad (5)$$

The mean of $S_{1,\Delta}$ is 0.5 and the standard deviation is .2041.

b. Sum of Two Variables ($S_{2,\Delta}$)

This random variable ($S_{2,\Delta}$) represents the vulnerable area for the fairly trivial case of an aircraft with only two critical components. Coding of realistic values as mentioned before has been used. $S_{2,\Delta}$ is the sum of two random variables, each drawn from identical symmetric, triangular distributions (0 to 1). The sum ($S_{2,\Delta}$) is defined over the interval 0 to 2. The cumulative is shown below.

$$F_{2,\Delta}(A_v) = 2/3(A_v)^4 \qquad , 0 \leq A_v \leq 1/2$$

$$F_{2,\Delta}(A_v) = -1/6 + 4/3\,A_v - 4(A_v)^2 + 16/3(A_v)^3 - 2(A_v)^4 \qquad , 1/2 \leq A_v \leq 1.0$$

$$F_{2,\Delta}(A_v) = 1 - (-1/6 + 4/3(2-A_v) - 4(2-A_v)^2 + 16/3(2-A_v)^3 - 2(2-A_v)^4) \qquad , 1 \leq A_v \leq 3/2$$

$$F_{2,\Delta}(A_v) = 1 - 2/3\,(2-A_v)^4 \qquad , 3/2 \leq A_v \leq 2$$

The mean of $S_{2,\Delta}$ is 1 and the standard deviation is .2887.

c. Sum of Twelve Variables ($S_{12,u}$)

This random variable ($S_{12,u}$) represents the vulnerable area (in coded form) of an aircraft with twelve critical components. $S_{12,u}$ is the sum of twelve random variables each drawn from identical uniform distributions (0 to 1). The sum is defined over the interval 0 to 12. Reference 3 shows some actual cumulative values for $S_{12,u}$ which will be presented in the next section.

Due to their complexity, the equations for this cumulative were not derived.

The mean of $S_{12,u}$ is 6 and the standard deviation is 1.

4. Comparison of Non-Normal Mathematically Derived Cumulatives and Their Normal Approximations

a. Definition of Terms

To aid the reader, the following definitions of terms are presented.

Since tables of the normal cumulative are given for mean 0 and standard deviation 1, a transformation is used for all variables to facilitate comparisons (random variables in caps, specific values in small case).

1. X is a random variable from a distribution with a mean of μ and standard deviation of σ. X can be standardized (transformed to Z) by the following formula:

$$Z = \frac{X - \mu}{\sigma}$$

Z has mean of 0 and a standard deviation of 1. Specific values of the random variable X are transformed to specific values of Z via the same concept.

$$z = \frac{x - \mu}{\sigma}$$

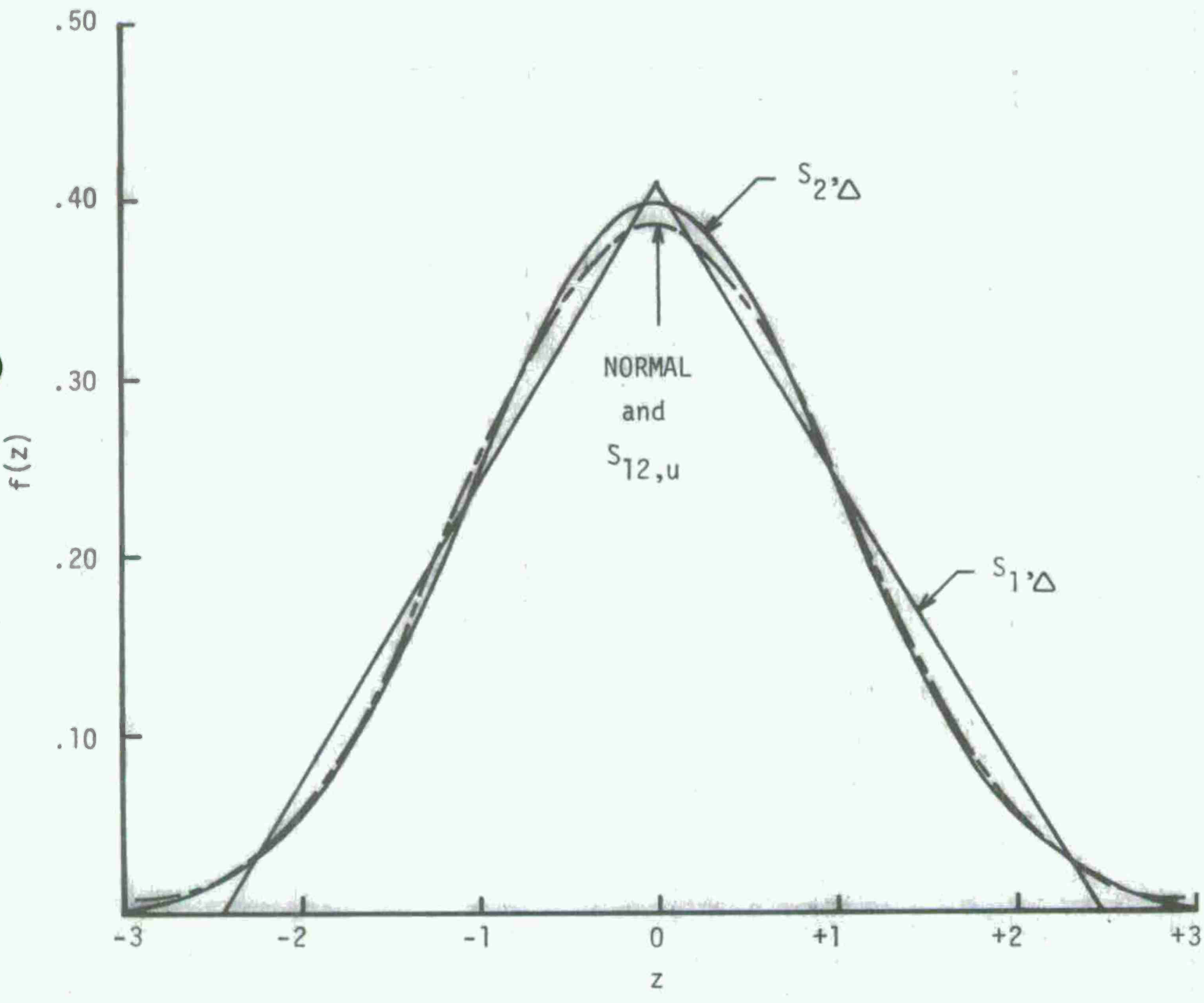
$\Phi(z)$ will be used to denote the cumulative distribution function for a <u>normally</u> distributed variable which has been standardized.

$F_{12,u}(z)$ will be used similarly to $\Phi(z)$ except that it will be used for <u>non-normal</u> distributions and the subscripts shown refer to the sum of twelve random variables each from identical, uniform distributions between 0 and 1.

b. <u>$S_{1,\Delta}$ vs Normal</u>

The probability density functions for the standardized $S_{1,\Delta}$ (from the exact analytic function) and standardized normal random variables (from a table) are shown in Figure 7 and their cumulatives are shown in Figure 8. For various z values, cumulative distribution values for the standardized $S_{1,\Delta}$ and standardized normal, and their differences are shown in Table 1. The maximum tabled difference of .0164 occurs at $|z| = 1.0$. Because of symmetry, positive z values need not be examined.

PROBABILITY DENSITY FUNCTIONS
FOR STANDARDIZED $S_{1,\Delta}$, $S_{2,\Delta}$ AND NORMAL



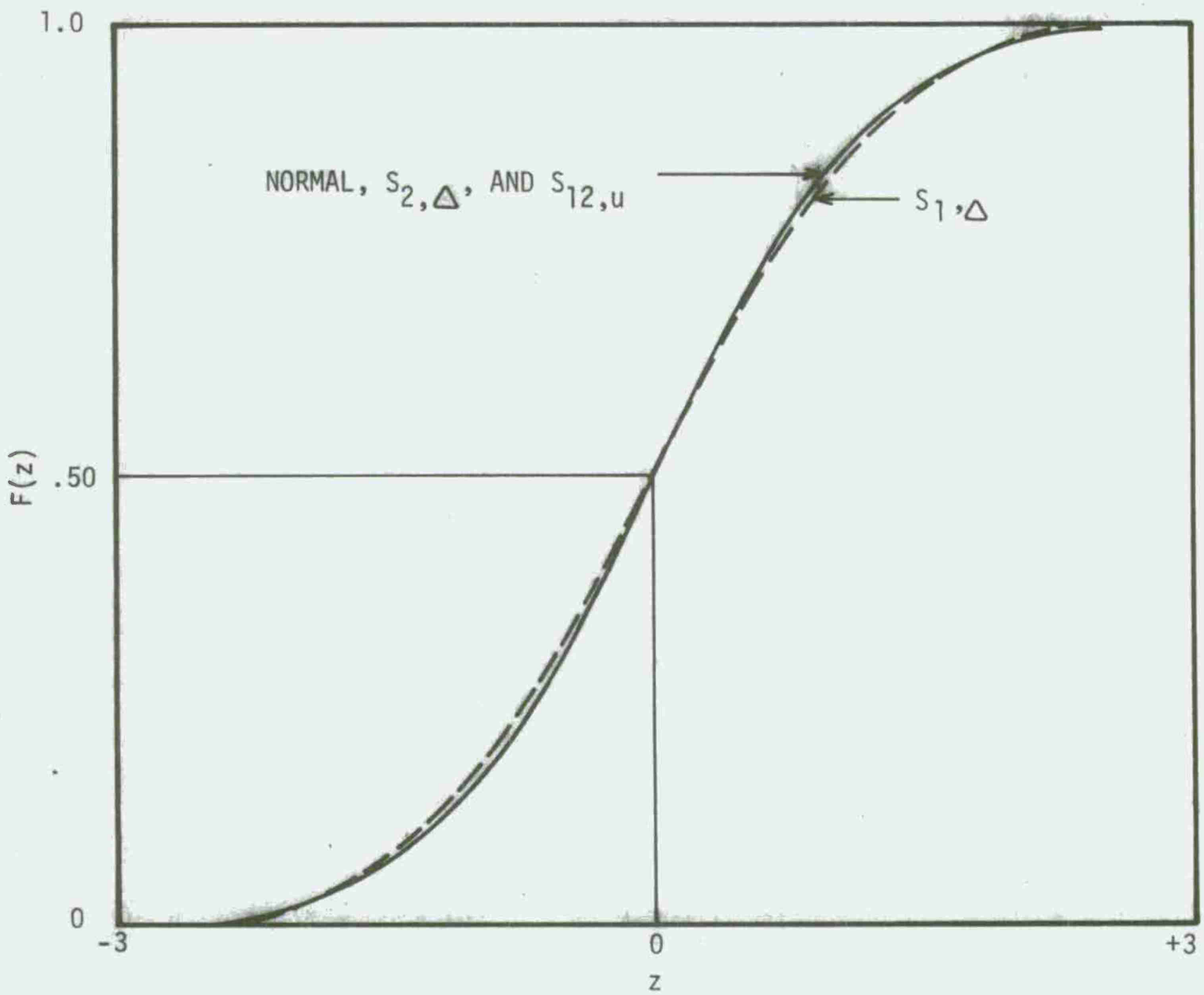
$S_{1,\Delta}$ - SYMMETRIC TRIANGULAR DISTRIBUTION

$S_{2,\Delta}$ - SUMMATION OF TWO IDENTICAL SYMMETRIC TRIANGULAR DISTRIBUTIONS

$S_{12,u}$ - SUMMATION OF 12 IDENTICAL UNIFORM DISTRIBUTIONS

FIGURE 7

CUMULATIVE DENSITY FUNCTIONS FOR STANDARDIZED $S_{1,\Delta}$, $S_{2,\Delta}$, NORMAL



$S_{1,\Delta}$ - SYMMETRIC TRIANGULAR DISTRIBUTION

$S_{2,\Delta}$ - SUMMATION OF TWO IDENTICAL SYMMETRIC TRIANGULAR DISTRIBUTIONS

FIGURE 8

COMPARISON OF $S_{1,\Delta}$ AND NORMAL

Standardized Cumulative

z	$S_{1,\Delta}$	$F_{1,\Delta}(z)$	Normal $\Phi(z)$	Difference $F_{1,\Delta}(z) - \Phi(z)$
-2.3	.0305	.0019	.0107	-.0088
-2.0	.0918	.0169	.0228	-.0059
-1.8	.1326	.0352	.0359	-.0007
-1.5	.1938	.0751	.0668	+.0083
-1.3	.2346	.1101	.0968	+.0133
-1.0	.2959	.1751	.1587	+.0164
-.8	.3367	.2267	.2119	+.0148
-.5	.3979	.3167	.3085	+.0082
-.3	.4388	.3851	.3821	+.0030
0	.5000	.5000	.5000	.0000

For $S_{1,\Delta}$: $\mu = .5$, $\sigma = .2041$

$$z_{1,\Delta} = \frac{S_{1,\Delta} - .5}{.2041}$$

TABLE 1

The next concept involves transformation of the approximation errors into the A_v dimension.

Table 2 shows the number of standard deviations required to include a certain percent of the $S_{1,\Delta}$ and normal distributions. Thus, for the

95 percent interval, use of the normal approximation would only be in error by (1.960 - 1.902) or .058 times the standard deviation. As the amount included approaches 100 percent, the normal approximation error increases. In statistical terminology, use of the normal approximation to estimate the limits which contain a specified percent of the non-normal distribution is said to be "robust" in that the approximation errors are small for many distributions which are non-normal (skewed, bi-modal, etc.).

Example:

$$\mu(A_v) = 100, \ \sigma(A_v) = 10$$

Distribution	95% Interval
$S_{1,\Delta}$	80.98 - 119.02*
Normal	80.40 - 119.60**

* $100 \pm 1.902(10)$

** $100 \pm 1.960(10)$

$S_{1,\Delta}$ VS NORMAL

Percent of the Distribution to be Included	Number of Standard Deviations To Include Percent in First Column	
	$S_{1,\Delta}$	Normal
80	$\pm$ 1.354	$\pm$ 1.282
90	$\pm$ 1.675	$\pm$ 1.645
95	$\pm$ 1.902	$\pm$ 1.960
98	$\pm$ 2.103	$\pm$ 2.326
99	$\pm$ 2.205	$\pm$ 2.576
99.9	$\pm$ 2.372	$\pm$ 3.291

TABLE 2

c. $S_{2,\Delta}$ VS Normal

$S_{2,\Delta}$ is the sum of two random variables, each from identical symmetric triangular distributions. The probability density for the standardized $S_{2,\Delta}$ can be compared to the standardized normal by inspection of Figure 7. The cumulative of the standardized $S_{2,\Delta}$ is so close to the standardized normal that it cannot be distinguished from the plotted normal in Figure 8. The values for $S_{2,\Delta}$ are from exact analytic expressions and the normal values are from tables.

Comparisons for various values of z are presented in Table 3. The maximum difference in the table (at $|z| = 0.8$) is .0073. Because of symmetry, positive z values need not be examined.

COMPARISON OF $S_{2,\Delta}$ AND NORMAL

Standardized Cumulative

z	$S_{2,\Delta}$	$F_{2,\Delta}(z)$	Normal $\Phi(z)$	Difference $F_{2,\Delta}(z) - \Phi(z)$
-2.5	.2782	.0040	.0062	-.0022
-2.3	.3360	.0085	.0107	-.0022
-2.0	.4226	.0213	.0228	-.0015
-1.8	.4803	.0355	.0359	-.0004
-1.5	.5670	.0688	.0688	+.0020
-1.3	.6247	.1009	.0968	+.0041
-1.0	.7113	.1653	.1587	+.0066
-.8	.7690	.2192	.2119	+.0073
-.5	.8556	.3147	.3085	+.0062
-.3	.9134	.3861	.3821	+.0040
0	1.0000	.5000	.5000	.0000

For $S_{2,\Delta}$: $\mu = 1.0$, $\sigma = .2887$

$$z_{2,\Delta} = \frac{S_{2,\Delta} - 1.0}{.2887}$$

TABLE 3

Table 4 shows the number of standard deviations required to include a certain percent of the $S_{2,\Delta}$ and normal distributions. For the 95 percent interval, use of the normal approximation would be in error by .020 times the standard deviation, which is roughly 1/3 of the error associated with the normal approximation for $S_{1,\Delta}$.

Example:

$$\mu(A_v) = 100, \ \sigma(A_v) = 10$$

Distribution	95% Interval
$S_{2,\Delta}$	80.60 - 119.40*
Normal	80.40 - 119.60**

* $100 \pm 1.940(10)$

** $100 \pm 1.960(10)$

$S_{2,\Delta}$ VS NORMAL

Percent of the Distribution to be Included	Number of Standard Deviations to Include the Percent in First Column	
	$S_{2,\Delta}$	Normal
80	$\pm$ 1.305	$\pm$ 1.282
90	$\pm$ 1.651	$\pm$ 1.645
95	$\pm$ 1.940	$\pm$ 1.960
98	$\pm$ 2.252	$\pm$ 2.326
99	$\pm$ 2.444	$\pm$ 2.576
99.9	$\pm$ 2.891	$\pm$ 3.291

TABLE 4

d. $S_{12,u}$ vs Normal

The cumulative distribution values for the standardized $S_{12,u}$ (obtained from reference 3), the standardized normal and their differences for various z values are shown in Table 5. The standardized $S_{12,u}$ is so close to the standardized normal that the difference is not distinguishable in either Figure 7 or Figure 8. The values for $S_{12,u}$ are from exact analytic expressions and the normal values are from tables.

The maximum tabled error is .0023 (at $|z| = 0.8$). Because of symmetry, positive z values need not be examined.

COMPARISON OF $S_{12,u}$ AND NORMAL

Standardized Cumulative

z	$S_{12,u}$	$F_{12,u}(z)$*	Normal $\Phi(z)$	Difference $F_{12,u}(z)-\Phi(z)$
-3.0	3.0	.0010	.0013	-.0003
-2.8	3.2	.0021	.0026	-.0005
-2.6	3.4	.0041	.0047	-.0006
-2.4	3.6	.0075	.0082	-.0007
-2.2	3.8	.0133	.0139	-.0006
-2.0	4.0	.0223	.0227	-.0004
-1.8	4.2	.0358	.0359	-.0001
-1.6	4.4	.0551	.0548	+.0003
-1.4	4.6	.0817	.0808	+.0009
-1.2	4.8	.1166	.1151	+.0015
-1.0	5.0	.1607	.1587	+.0020
-.8	5.2	.2142	.2119	+.0023
-.6	5.4	.2765	.2743	+.0022
-.4	5.6	.3463	.3446	+.0017
-.2	5.8	.4217	.4207	+.0010
0	6.0	.5000	.5000	.0000

For $S_{12,u}$: $\mu = 6.0$, $\sigma = 1.0$

$$z_{12,u} = \frac{S_{12,u} - 6.0}{1.0}$$

*Obtained from reference 3.

TABLE 5

Table 6 shows the number of standard deviations required to include a given percentage of the $S_{12,u}$ and normal distributions. For the 95 percent interval, use of the normal approximation would be in error by .0089 times the standard deviation, which is less than 1/2 of the error associated with the normal approximation for $S_{2,\Delta}$.

Example:

$$\mu(A_v) = 100, \ \sigma(A_v) = 10$$

Distribution	95.54% Interval
$S_{12,u}$	80.000 - 120.000*
Normal	79.911 - 120.089**

* 100 $\pm$ 2.0000(10)

** 100 $\pm$ 2.0089(10)

An inference from these three comparisons is that, as the number of random variables, P(K/H)'s, increases from 1 to 12, the error caused by using the normal approximation decreases and the approximation can be used to include a greater percent of the distribution with a specified error.

$S_{12,u}$ VS NORMAL

Percent of the Distribution to be Included	Number of Standard Deviations to Include the Percent in First Column	
	$S_{12,u}$*	Normal
76.68	± 1.2000	± 1.1912
83.66	± 1.4000	± 1.3937
88.97	± 1.6000	± 1.5969
92.84	± 1.8000	± 1.8018
95.54	± 2.0000	± 2.0089
97.35	± 2.2000	± 2.2183
98.50	± 2.4000	± 2.4304
99.80	± 3.0000	± 3.0882
99.998	± 4.0000	± 4.3004

*From reference 3.

TABLE 6

5. Extensions of Normal Approximation to Other Non-Normal Distributions

Further study of the Central Limit Theorem and its strengths (reference 2) reveals that:

(1) The distributions of the variables to be summed need not be symmetric.

(2) The more closely each summand approximates a normal, the fewer summands needed for the sum to approximate normal.

(3) The distribution of the sum is exactly normal if the summand distributions are normal.

The Central Limit Theorem in its most general form also shows that:

(1) Summands need not be identically distributed.

(2) Some relaxation of the independence of the summand distributions is possible.

a. $\underline{S_{12,\Delta}}$

$S_{12,\Delta}$ is the sum of twelve random variables each drawn from identical symmetric triangular distributions. The triangular distribution is closer to the normal than the uniform. Therefore, it can be concluded that the use of the normal approximation for $S_{12,\Delta}$ has less error than the normal approximation for $S_{12,u}$.

b. $\underline{S_{(1,\Delta + 1,N)}}$

$S_{(1,\Delta + 1,N)}$ is the sum of two random variables, one from a symmetric triangular and one from a normal distribution. An assumption must be made for extension of the Central Limit Theorem to this case -- the means of the two distributions are approximately equal and the standard deviations are approximately equal. If the mean of the triangular distribution were considerably larger than the normal, the sum would be very close to that of a triangular distribution (the effect of the normal portion of the sum would be insignificant).

Thus, it can be concluded for the above assumption that the sum of a triangular and normal is closer to normal than the sum of two identical triangular distributions.

c. $\underline{S_{12,\text{ mixed }\Delta,u}}$

$S_{12,\text{ mixed }\Delta,u}$ is the sum of twelve random variables each from either a symmetric triangular or a uniform. Again, each of the twelve distributions is assumed to have the same mean and variance.

Since the triangular is closer to normal than uniform, it can be concluded that $S_{12,\text{ mixed }\Delta,u}$ is closer to normal than $S_{12,u}$. If the mix contains one or more normals, then that sum is closer to normal than the sum which contains only triangular and uniform.

Furthermore, the sum containing only triangular ($S_{12,\Delta}$) will be closer to normal than the sum containing a mix of triangular and uniform.

d. Non-Symmetric Distributions

Since symmetric distributions are closer to normal than non-symmetric, sums containing only symmetric distributions approximate normality with fewer summands than sums containing both symmetric and non-symmetric (from reference 2).

6. Departure of A_V From Assumptions of the Central Limit Theorem

In the foregoing discussions, all of the variables, $P'(K/H)_i$, were distributed between 0 and 1. For components having uniform and triangular distributions, equation (7) is a more realistic expression for A_V.

$$A_V = \sum_{i=1}^{n} [A_i(U_i - L_i)P'(K/H)_i + L_i] \tag{7}$$

For distributions other than uniform and triangular, similar transformations are available but are not presented in this report. The variable $P'(K/H)_i$ has a coefficient of $A_i(U_i - L_i)$. If these coefficients are not equal for all components, the Central Limit Theorem assumption of identically distributed variables is not met; hence, the normality approximation may not be as good.

An extreme example of unequal coefficients (presented areas) is shown in the following equation which represents a target with 12 critical components.

$$A_v = 10{,}000\ X_1 + X_2 + X_3 + \ldots + X_{12}\ , \tag{7a}$$

where the X_i's are uniformly distributed between zero and one.

A Monte Carlo simulation (techniques described in Section III) was performed and data points from a sample of 1,000 values of A_v from equation (7a) are shown in Figures 9 and 10. Figure 9 shows the negative z values and Figure 10 shows the positive z values. Figures 9 and 10 also show data points from the simulation of $S_{12,u}$ (sample of 1,000), the exact analytically derived values for the uniform ($S_{1,u}$) and the normal (table values). $S_{12,u}$ is the same variable as shown in equation (7a) except that all coefficients of $S_{12,u}$ equal one. All four variables shown in the graphs are in standardized form (z). Not all of the 2,000 simulated data points are plotted; those points omitted follow the trends that are shown.

Examination of Figures 9 and 10 shows that A_v from equation (7a) is close to the uniform while the simulation of $S_{12,u}$ is close to the normal. This shows that the shape of A_v from equation (7a) is dominated by the term, 10,000 X_1; however, both equations (7a) and $S_{12,u}$ are quite close to normal.

Table 7 lists specific values of the variables that are plotted in Figures 9 and 10.

CUMULATIVE DISTRIBUTION DATA POINTS FROM SIMULATIONS OF EQUATION (7a) AND $S_{12,u}$ FOR COMPARISON TO ANALYTIC NORMAL AND UNIFORM (NEGATIVE VALUES OF z)

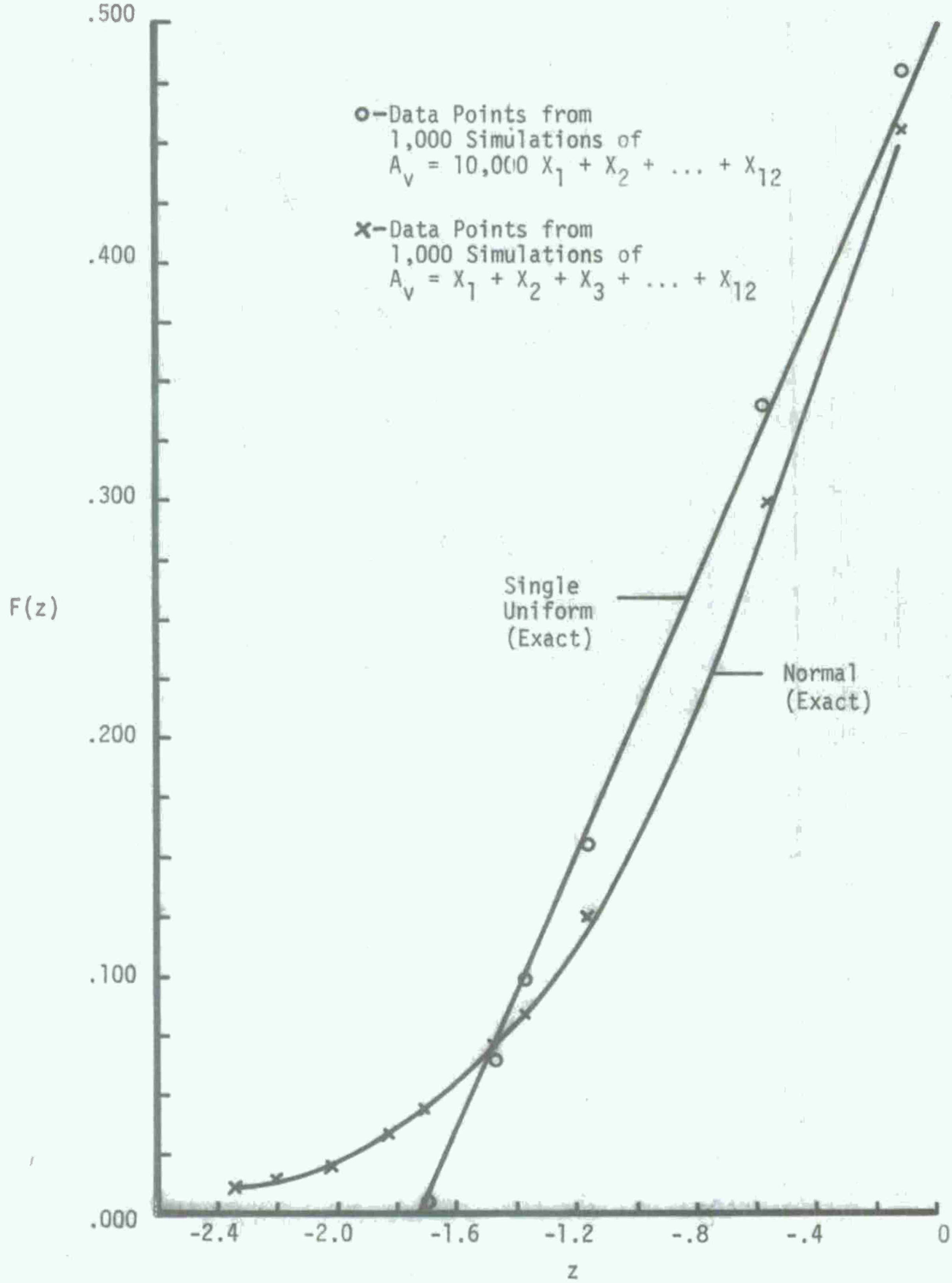


FIGURE 9

CUMULATIVE DISTRIBUTION DATA POINTS FROM SIMULATIONS OF EQUATION (7a) AND $S_{12,u}$ FOR COMPARISON TO ANALYTIC NORMAL AND UNIFORM (POSITIVE VALUES OF z)

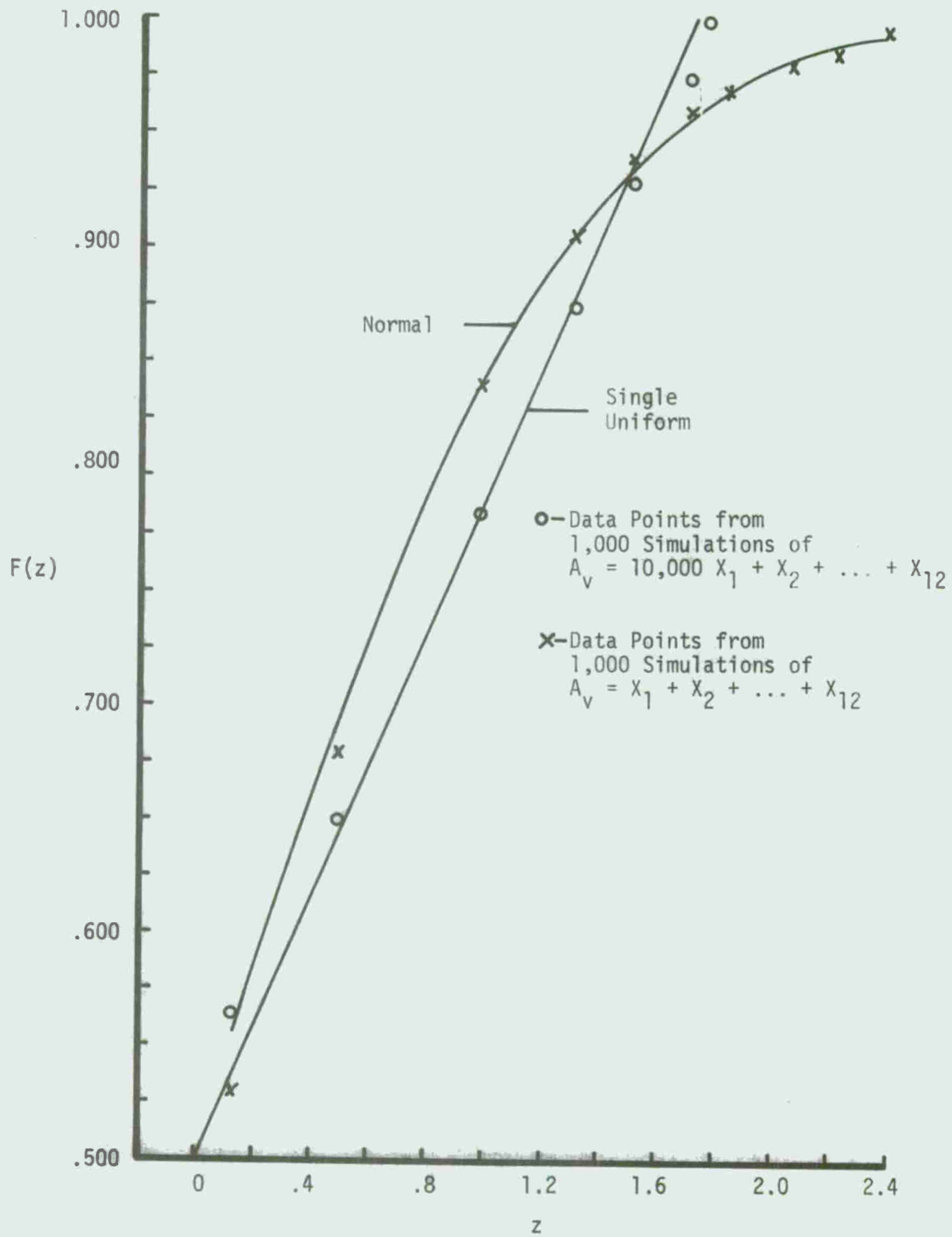


FIGURE 10

CUMULATIVE VALUES

z	Simulated Equation (7a)*	Mathematically Derived Single Uniform ($S_{1,u}$)	Simulated Sum of 12 Uniform** ($S_{12,u}$)	Normal (From Tables)
-2.33	.000	.000	.010	.010
-2.20	.000	.000	.015	.014
-2.04	.000	.000	.020	.021
-1.83	.000	.000	.035	.034
-1.73	.000	.000	***	.042
-1.70	.005	.009	.045	.045
-1.48	.065	.073	.070	.069
-1.37	.100	.105	.085	.085
-1.16	.155	.165	.125	.123
- .57	.340	.335	.300	.284
- .11	.480	.468	.455	.456
+ .13	.565	.538	.530	.552
+ .50	.650	.644	.680	.692
+ .99	.785	.786	.840	.839
+1.31	.875	.878	.905	.905
+1.52	.930	.939	.940	.936
+1.71	.975	.994	.960	.956
+1.73	***	1.000	***	.958
+1.78	1.000	1.000	***	.963
+1.84	1.000	1.000	.970	.967
+2.06	1.000	1.000	.980	.980
+2.21	1.000	1.000	.985	.986
+2.39	1.000	1.000	.995	.992

* $A_v = 10{,}000\ X_1 + X_2 + X_3 + \ldots + X_{12}$

** $A_v = X_1 + X_2 + X_3 + \ldots + X_{12}$

*** Exact z value did not occur in simulation.

TABLE 7

Some working rules must be developed to assess the changes in number of components, the probability density form of $P(K/H)_i$ for each component, the upper and lower limits of $P(K/H)_i$ for each component, and the presented area (A_i) of each component. Some examples of these combinations are shown below.

Case I. One component's area very large compared to the others.

Set n = 101; all P(K/H)'s symmetric triangularly distributed.

$$(U_1 - L_1) = (U_2 - L_2) = \ldots = (U_{101} - L_{101}) = 0.1$$

$$A_1 = A_2 = \ldots = A_{100} = 1 \text{ ft}^2; \; A_{101} = 100 \text{ ft}^2$$

$$A_v = \sum_{i=1}^{100} [(1)\ (0.1)\ P'(K/H)_i + L_i] + 100[(0.1)\ P'(K/H)_i + L_{101}], \text{ or}$$

$$A_v = \underbrace{[(0.1) \sum_{i=1}^{100} P'(K/H)_i]}_{\text{Part (A)}} + \underbrace{[10\ P'(K/H)_{101}]}_{\text{Part (B)}} + \sum_{1}^{100} L_i + 100\ L_{101} \tag{8}$$

Part (A) of Equation (8) may be considered as normally distributed with expectation 5.0 and $\sigma^2 = 100/24$. Part (B) of Equation (8) is a triangular distribution with expectation 5.0 and $\sigma^2 = 100/24$. Therefore, Case I reverts to the sum of a normal and a triangular, which is nearer normal than the sum of two triangulars.

Case II. Two components with very large relative areas.

n = 10; all P(K/H)'s are from the same triangular distribution.

$$(U_1 - L_1) = (U_2 - L_2) = \ldots = (U_{10} - L_{10}) = 0.1$$

$$A_1 = A_2 = \ldots = A_8 = 1 \text{ ft}^2; \; A_9 = A_{10} = 100 \text{ ft}^2$$

$$A_v = \underbrace{\sum_{i=1}^{8} [(1)\ (0.1)\ P'(K/H)_i]}_{\text{Part (A)}} + \underbrace{100(.1)P'(K/H)_9}_{\text{Part (B)}}$$

$$+ \underbrace{100(.1)P'(K/H)_{10}}_{\text{Part (C)}} + \sum_{i=1}^{10} A_i L_i \tag{9}$$

The sum of Parts (B) and (C) of Equation (9) reverts to the sum of two identically distributed triangulars, which was previously shown to be near normal. Part (A) will have a further normalizing effect, although this effect is somewhat weakened by the fact that the expected value of Part (A) is roughly 1/12 of the expected value of either Part (B) or Part (C).

<u>Other Cases</u>

If all of the $(U_i - L_i)$'s are not equal, then this will disturb the normality to some degree. For example, if $(U_1 - L_1) = 10(U_2 - L_2)$, then this effect will be the same as that for $A_1 = 10\ A_2$.

Further analysis will be performed using realistic values for $P(K/H)_i$ and A_i to substantiate a set of working rules.

D. <u>Methodology (A_v Approximated by Normal)</u>

If A_v can be considered normally distributed, the procedure for determining the limits of A_v that contain a specific percent of all A_v values involves the following steps:

(1) Find z from the cumulative normal tables, z vs $\phi(z)$, such that the specified percent is within $\pm z$.

(2) Calculate the mean and standard deviation of A_v as shown in Equations (10) through (17).

(3) Transform +z to A_v upper limit and -z to A_v lower limit by the formula

$$z = \frac{A_v - \mu(A_v)}{\sigma(A_v)} \text{ or } A_v = z\sigma(A_v) + \mu(A_v)\ .$$

1. Calculation of Mean

Using the algebra of expectations, the expected value of A_v is calculated as follows:

$$E(A_v) = \mu(A_v) = E(\sum_{i=1}^{n} A_i P(K/H)_i)$$

$$= \sum_{i=1}^{n} E(A_i\ P(K/H)_i)$$

$$= \sum_{i=1}^{n} A_i\ E(P(K/H)_i) \qquad (10)$$

For the uniform distribution,

$$E(P(K/H)_i) = .5(U_i + L_i) \qquad (11)$$

The general triangular distribution is depicted in Figure 11.

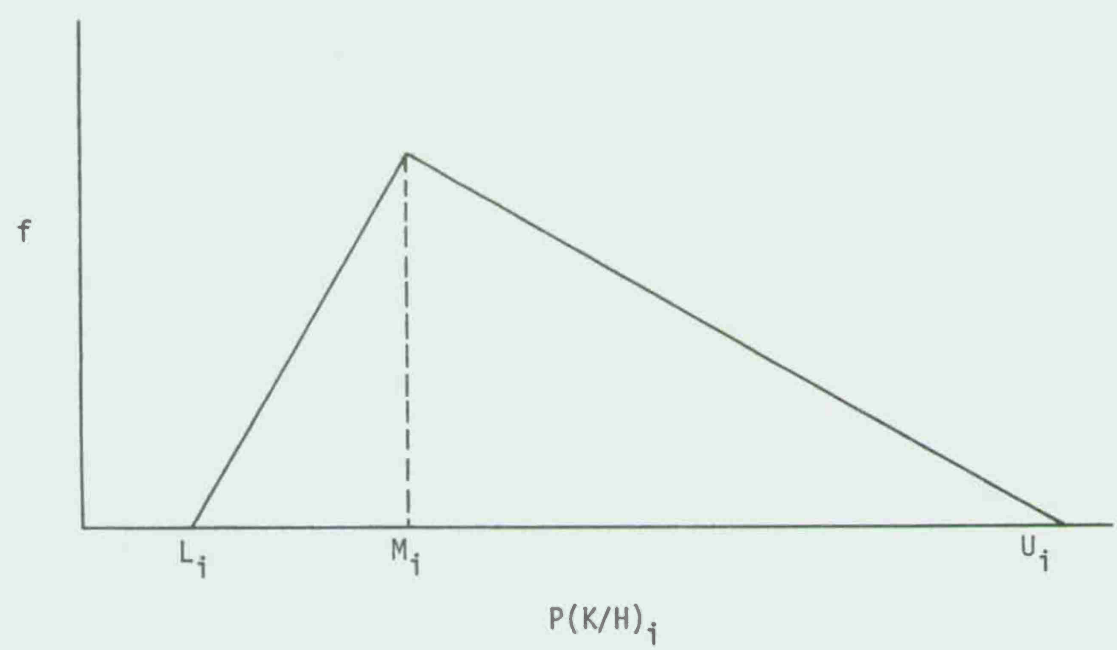


FIGURE 11

The distribution is defined by specifying M_i, L_i, U_i, and the expected value is given by

$$E(P(K/H)_i) = 1/3\ (U_i + L_i + M_i)\ . \tag{12}$$

For a normal distribution,

$$E(P(K/H)_i) = \mu_i ; \tag{13}$$

$\overline{X}_i$ from a sample can be used as an estimator of μ_i, $\overline{X}_i = \sum_{j=1}^{m} X_{ij}/m$, where m is the sample size.

2. Calculation of Standard Deviation

The standard deviation of A_v is calculated as follows:

$$\sigma(A_v) = [\sum_{i=1}^{n} \mathrm{Var}\ (A_i\ P(K/H)_i)]^{1/2}$$

$$= [\sum_{i=1}^{n} A_i^2\ \mathrm{Var}\ (P(K/H)_i)]^{1/2}\ . \tag{14}$$

The variance of individual components is calculated as follows:

For the normal distribution,

$$\mathrm{Var}\ (P(K/H)_i) = \sigma_i^2 ; \tag{15}$$

s_i^2 from a sample can be used as an estimator of σ_i^2, $s_i^2 = \dfrac{m\left[\sum_{j=1}^{m} X_{ij}^2\right] - \left[\sum_{j=1}^{m} X_{ij}\right]^2}{m(m-1)}$ where m is the sample size.

For the uniform distribution,

$$\mathrm{Var}\ ((P(K/H)_i) = \frac{(U_i - L_i)^2}{12}\ . \tag{16}$$

For the triangular distribution,

$$\mathrm{Var}(P(K/H)_i) = 1/18[(M_i-L_i)^2 + (M_i-L_i)(U_i-M_i) + (U_i-M_i)^2] \tag{17}$$

III. MONTE CARLO SIMULATION DESCRIPTION

The Monte Carlo simulation for the cumulative distribution of A_V was developed and test case answers were checked to verify computer code structure, under the following conditions:

a. The distribution form of $P(K/H)_i$ can be triangular, uniform, or normal. Other distribution forms can be added later.

b. Distribution parameters must be specified as follows:

(1) Uniform

L_i = the lower limit of $P(K/H)_i$

U_i = the upper limit of $P(K/H)_i$

(2) Triangular

L_i = the lower limit of $P(K/H)_i$

U_i = the upper limit of $P(K/H)_i$

M_i = modal (largest frequency) value of $P(K/H)_i$

(3) Normal

Mean (μ_i) and standard deviation (σ_i)

c. Presented area (A_i) of each component must be specified.

d. The number of sample values of A_V desired is specified as K.

The following transformations of variables were performed to assist in the ease of programming.

a. Uniform

The transformation made for this distribution and the probability density functions of both variables are shown below.

$$P(K/H)_i = (U_i - L_i)\ P'(K/H)_i + L_i \tag{18}$$

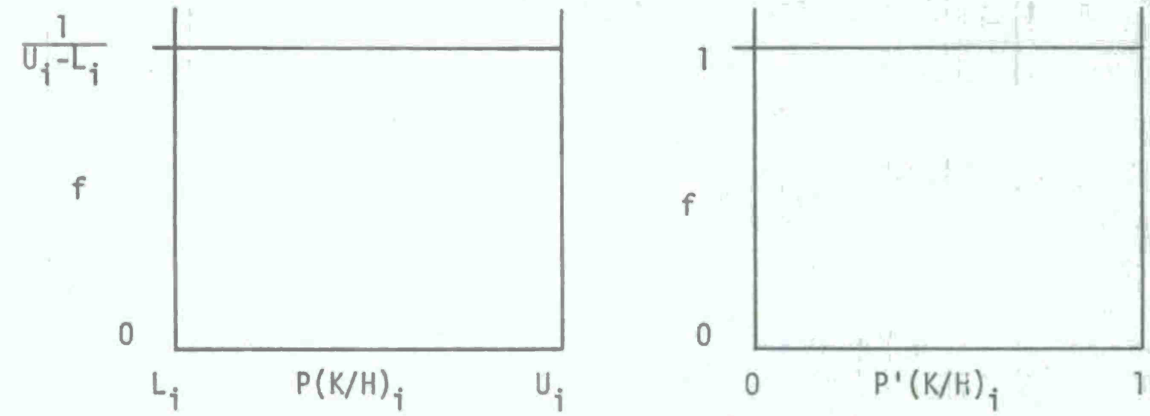


FIGURE 12

The cumulative distribution for $P'(K/H)_i$ is

$$F_{1,u}(P'(K/H)_i) = \int_0^1 f(P'(K/H)_i)\, d(P'(K/H)_i) \qquad (19)$$

This distribution is shown in graphical form in Figure 13.

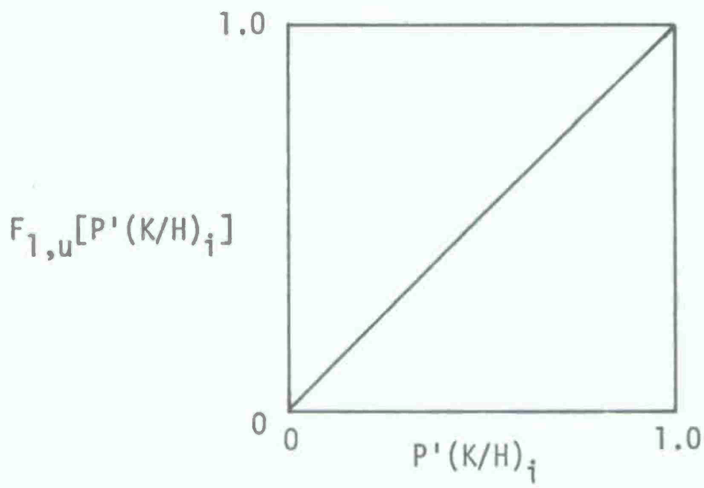


FIGURE 13

Thus, $P'(K/H)_i = F_{1,u}[P'(K/H)_i]$, which means that a uniform random number [0,1] is drawn, equated to $P'(K/H)_i$, and then transformed to a value of $P(K/H)_i$.

b. Triangular

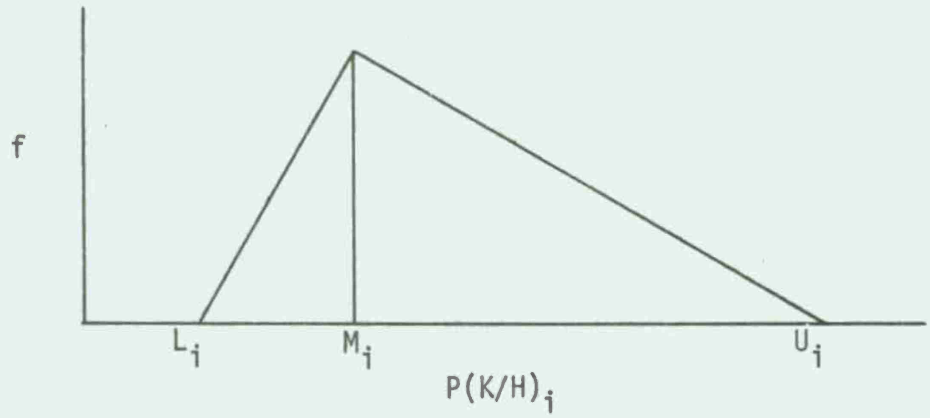


FIGURE 14

A uniform (0,1) random number is drawn and equated to $F[P(K/H)_i]$. If $F[P(K/H)_i] \leq (M_i-L_i)/U_i-L_i)$, then

$$P(K/H)_i = L_i + \sqrt{F[P(K/H)_i](M_i-L_i)(U_i-L_i)} \tag{20}$$

If $F[P(K/H)_i] \geq (M_i-L_i)/(U_i-L_i)$, then

$$P(K/H)_i = U_i - \sqrt{(U_i-M_i)(U_i-L_i)(1-F[P(K/H)_i])} \tag{21}$$

c. Normal

One method used on computers to approximate the normal is to sum twelve random numbers each drawn from a uniform [0,1] distribution [No simple explicit means exists to calculate X given $\Phi(X)$].

$$S_{12,u} = \sum_{j=1}^{12} (RN)_j \tag{22}$$

$S_{12,u}$ has an expectation of 6 and standard deviation of one; therefore,

$$P(K/H)_i = (S_{12,u} - 6.0)\ \sigma_i + \mu_i \tag{23}$$

which has an expectation of μ_i and standard deviation of σ_i.

The values of A_v generated in the simulation are input to a general statistical program from the NWSC Crane program library. The output from this program includes the mean, standard deviation, skewness, kurtosis, frequency histogram, actual values of A_v, actual cumulative distribution values, theoretical normal cumulative distribution for each of the observed values of A_v, and the maximum deviation between observed cumulative and theoretical normal cumulative.

IV. MONTE CARLO SIMULATION RESULTS

A comparison of the mathematically derived cumulative distribution function and the simulated cumulative distribution function is needed to obtain an estimate of the error that might be present in the simulation of an unknown cumulative distribution function. Simulation results are shown in Table 8 for a single triangularly distributed variable X, and in Table 9 for $A_v = X_1 + X_2$, where X_1 and X_2 are both triangularly distributed. Also, the maximum errors in using normal approximations to the mathematically derived cumulative distributions are shown in these tables.

Simulation Results for a Single Triangularly Distributed Variable

Number of Replications	Computer Simulation Cost	Maximum Error in Cumulative Distributions: Simulated $F_{1,\Delta}(z)$ vs. Actual $F_{1,\Delta}(z)$	Normal, $\Phi(z)$ vs. Actual $F_{1,\Delta}(z)$
200	\$ 2.80	.034	.016
1000	\$11.60	.019	.016

TABLE 8

Simulation Results for the Sum of Two Triangularly Distributed Variables ($A_v = X_1 + X_2$)

Number of Replications	Computer Simulation Cost	Simulated $F_{2,\Delta}(z)$ vs. Actual $F_{2,\Delta}(z)$	Normal, $\Phi(z)$ vs. Actual $F_{2,\Delta}(z)$
200	\$ 2.87	.030	.007
1,000	11.79	.021	.007
10,000	320.75	.002	.007

TABLE 9

Table 9 shows that for 200 replications of the simulated $A_v(S_{2,\Delta})$, the maximum observed error in the cumulative was over 4 times that of the normal approximation. For 1,000 replications, the ratio was 3 to 1, while for 10,000 replications, the simulation error was less than 1/3 of the normal approximation error (see simulation results). The simulation error for 10,000 replicates of the vulnerable area of an aircraft with two critical components is very small; however, the computer simulation cost of \$320 seems prohibitive.

It can also be noted from observation of Tables 8 and 9 that the error inherent in the Monte Carlo simulation is sensitive to the number of replications as well as the number of summands. The error in the normal approximation, on the other hand, is sensitive only to the number of summands. As the number of summands increases, the normal will more closely approximate the actual distribution.

V. CONCLUSIONS

1. Considering the trade-off between accuracy and cost, the normal approximation is the appropriate technique for calculating the assurance limits for A_v. This approximation should be used to compute the upper and lower limits such that there is 95% assurance that A_v will be between these limits (95% is used as an example). The steps that are used to calculate these limits are as follows:

Step 1 - Compute $\overline{X}(A_v)$.

Step 2 - Compute $s(A_v)$.

(NOTE: Symbols $\overline{X}$, s are used since μ, σ will probably have to be estimated.)

Step 3 - Find z in the table to give desired assurance.

% Assurance	z
80	1.282
90	1.645
95	1.960
98	2.326
99	2.576
99.8	3.090

Step 4 - Compute Assurance Limits

Upper Limit, $\overline{X}(A_v) + zs(A_v)$

Lower Limit, $\overline{X}(A_v) - zs(A_v)$

2. The accuracy of the normal approximation will increase as the number of critical components in the vulnerability analysis increases. An example of the normal approximation error may be seen by examining the following table.

Comparison of the exact (mathematically derived) assurance limits versus the limits assuming normality shows closer agreement as the number of critical components increases from 1 to 2 to 12 ($S_{1,\Delta}$ to $S_{2,\Delta}$ to $S_{12,u}$).

Example

$$\overline{X}(A_v) = 100 \text{ ft}^2, \ s(A_v) = 10 \text{ ft}^2$$

Distribution	95% Assurance for $A_v(ft^2)$ Range	Limits
$S_{1,\Delta}$	± 19.02	80.98 - 119.02
$S_{2,\Delta}$	± 19.40	80.60 - 119.40
Normal	± 19.60	80.40 - 119.60

Distribution	95.54%* Assurance for $A_v(ft^2)$ Range	Limits
$S_{12,u}$	± 20.000	80.000 - 120.000
Normal	± 20.089	79.911 - 120.089

* 95% limits were not readily available for $S_{12,u}$.

3. Logical extensions of statistical theory show that as the distributions of the P(K/H)'s become more nearly normal, A_v will become more normal, and that if the P(K/H)'s are exactly normal, then A_v is exactly normal. This progression toward normality is shown by the following ranking (from least normal toward normal);

a. 12 components, all distributions uniform

b. 12 components, mixed uniform and symmetric triangular distributions

c. 12 components, all distributions symmetric triangular

d. 12 components, mixed symmetric triangular and normal distributions

Thus, the assurance limits for A_v as described by (b), (c), or (d), above, would be closer to the normal than $S_{12,u}$ as shown in Conclusion 2.

4. In certain vulnerability analyses where one of the critical components is many times larger than any of the other critical components, the accuracy of the normal approximation is decreased but is still acceptable. For example, fuel tanks are major contributors (large presented area) in vulnerable area assessments of fixed wing aircraft.

5. There are an infinite number of exceptions to and deviations from the Central Limit Theorem and its assumptions that could be examined. The real strength of this normal approximation concept can be better examined when the presented area and P(K/H) distribution for each critical components of an actual aircraft are obtained.

6. After a successful demonstration of the assurance limits methodology on actual aircraft data, these techniques should be incorporated into the VAREA Program. These additions will involve minimal changes to the existing methodology and a negligible increase of the execution time.

7. The presented areas and point estimates of P(K/H) for critical components have received considerable study and should be well documented; however, availability of these values in published summary form is somewhat limited. The major work thrust that is needed concerns the determination of the distributions of P(K/H) values.

Specifically, these P(K/H) distributions can be estimated by analyzing sources of variability such as:

a. Drive shafts shear at different loads and control rods fail at different compression loads for a fixed amount of the shaft or rod removed.

b. Projectiles on a given shotline (same impact point) would remove various amounts of the shaft.

c. P(K/H) values for each of several aspect angles are pooled into one value.

d. Penetration (Thor) equations are not perfect predictors of residual mass and velocity.

e. Field test data such as the fraction of shots which render a component non-functional are subject to variability.

8. Continuing efforts will consist of quantifying the variability, determining some realistic distributions for P(K/H) values and developing methodology for handling the restrictions placed on distributions by the shotline survivor rule.

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